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## A STUDY ON TWO WAREHOUSE INVENTORY MODEL WITH STOCK DEPENDENT DEMAND AND SHORTAGES

Amit Kumar Attri<sup>\*1</sup> & S.R. Singhand Sachinkumar<sup>2</sup>

<sup>\*1&2</sup>Department of Mathematics, CCS University, Meerut, India

### ABSTRACT

Here in this paper an inventory model for deteriorating products with two warehouse storage capacity has been developed. It is observed many times that the demand for any product vary with the stock level. A high stock level attracts the customers to buy more but it is true only up to a certain level of stock and after that the demand becomes constant. Here a rented warehouse is used to stock the remaining quantity after filling the owned warehouse. The shortages are allowed in owned warehouse and occurring shortages are partially backlogged. The optimal cost of the system is illustrated with the help of a numerical example. The sensitivity analysis of the system with respect to different associated parameters is also presented to check the feasibility of the model.

**Keywords:** Warehouse, Inventory, Stock dependent demand, Shortages, Lost sale, Deterioration.

### 1. INTRODUCTION

Inventory of physical goods is a major part for any business organization since a huge part of money is tied with it comparative to others. So it is necessary to manage the inventory properly such that one can optimize the total cost/profit. Generally it is seen that a high stock level attracts the customers to buy more, but as well it increases the different associated cost such as purchasing and holding. So in this case one has to find that value of ordering quantity that can maximize the profit and minimize the cost. It was firstly pointed out by the Levin et al. (1972) that “the presence of inventory has a motivational effect on the people around it.” In the last several years many researchers worked on stock level dependent demand. Gupta and Vrat (1986) firstly worked on the inventory models with stock dependent demand rate. Mandal and Phaujdar (1989) presented an economic production quantity inventory model for deteriorating items with stock level dependent consumption rate. Datta and Pal (1998) developed an inventory model with the concept of demand promotion by upgradation under stock-dependent demand situation. Alfares (2007) introduced an inventory model with stock-level dependent demand rate and variable holding cost. Arya et al. (2009) presented an order level inventory model for perishable items with stock dependent demand and partial backlogging. Khurana et al. (2015) developed a supply chain production inventory model for deteriorating product with stock dependent demand under inflationary environment and partial backlogging. Singh et al. (2016) introduced an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Singh et al. (2016) also presented an inventory model for deteriorating items having seasonal and stock-dependent demand with allowable shortages.

The warehouse storage capacity is defined as the storage space needed to hold the stock. To assume that the products will be delivered exact at that time when it is required is impractical. So to consider the influence of warehouse capacity is a very important factor in the development of inventory models. Considering the constant rate of deterioration Sarma (1987) presented an inventory model for deteriorating items with infinite replenishment rate and shortages. Pakkala and Achary (1994) developed a two warehouse inventory model for deteriorating items with bulk release pattern. In these models, the demand rate was assumed to be constant function. Kar et al. (2001) introduced a fixed time horizon deterministic inventory model with two levels of storage facility under a linear trend in demand. Chung and Huang (2007) came forward with a two-warehouse inventory model for deteriorating items under allowable trade credit financing. Singh and Jain (2009) presented a deterministic inventory model over a finite planning horizon with variable rate of deterioration and a linear trend in demand. Singh et al. (2013) proposed a three stage supply chain model with two warehouse, imperfect production, variable demand rate and inflation. Tayal et al. (2014) introduced a deteriorating production inventory problem with space restriction. In this model, at the arrival of stock, the extra ordered quantity from the warehouse capacity is returned to the supplier, for which the supplier charges a penalty cost on the retailer. The assumption that the products maintain its quality during storage is not true for all the products. There are so many products which loss their quality due to breakage, evaporation, dryness, obsolescence etc. Then these



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products are not in perfect condition to be used perfectly. So to consider the effect of deterioration in the development of inventory model is very necessary. Ghare and Schrader (1963) were the first to consider the effect of deterioration in inventory modeling. Covert and Philip (1973) proposed an EOQ model for items with Weibull distribution deterioration. Kang and Kim (1983) studied on the price and production level of the deteriorating inventory system. Raafat et al. (1991) also developed an inventory model for deteriorating items. Patra (2010) presented an order level inventory model for deteriorating items with partial backlogging. Tayal et al. (2014) proposed a two echelon supply chain model for deteriorating items with effective investment in preservation technology to reduce the existing rate of deterioration. Singh et al. (2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages. Tayal et al. (2015) introduced an inventory model for deteriorating items with seasonal products and an option of an alternative market. After that, in 2015 Tayal et al developed an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate. Since there are many products in general life which maintain its quality for a certain period of time and after that it begin to deteriorate. Tayal et al. (2016) introduced an inventory model taking into consideration the effect of preservation technology for deteriorating products under trade credit period. Aljazzar et al (2017) investigated a coordination of a three-level supply chain (supplier–manufacturer–retailer) by coupling two well-known trade credit mechanisms that are widely used in practice, permissible delay in payments and price discounts, where the length of the delay period and discounts offered along the supply chain are decision variables.

Here in this paper we have proposed to derive a two warehouse optimal inventory policy for deteriorating items with stock dependent demand and partially backlogging of occurring shortages.

On the arrival of stock the remaining quantity after filling the owned warehouse is transferred to rented warehouse. First the inventory in rented house is used to satisfy the customer's demand and after that the inventory in owned warehouse is used. The purpose of this model is to find out the optimal total cost of the system. A numerical example and sensitivity analysis are presented to illustrate the model.

## 2. ASSUMPTIONS & NOTATIONS

### Assumptions:

- 1) The products assumed in this model are deteriorating in nature and deterioration rate is taken as a function of time and is given by  $\theta t$ .
- 2) The demand for the products is stock dependent.
- 3) The demand for the products is stock dependent up to a certain level and after that it is constant.

$$D = \begin{cases} \alpha + \beta W, & 0 \leq t \leq t_1 \\ \alpha + \beta I(t), & t_1 \leq t \leq v \\ \alpha, & v \leq t \leq T \end{cases} \quad \alpha, \beta > 0$$

- 4) The deteriorated units are completely discarded.
- 5) The lead time is not considered in the development of this model.
- 6) The shortages are allowed and partially backlogged at a constant rate.
- 7) The holding cost in R.W. is greater than the holding cost in O.W.
- 8) The O.W. has a fix capacity of  $W$  units and the R.W. has unlimited capacity.
- 9) The goods in O.W. is consumed only after the consumption of goods kept in R.W.

### Notations:

$\alpha, \beta$	demand parameters
$W$	owned warehouse capacity
$\theta$	deterioration parameter
$t_1$	the time at which the stock level in R.W. becomes zero.
$v$	the time at which the inventory level in O.W. becomes zero.
$T$	replenishment cycle
$I_r(t)$	inventory level at any time $t$ in R.W.



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$I_0(t)$	inventory level at any time $t$ in O.W.
$A$	ordering cost per order
$h_1$	holding cost per unit in R.W.
$h_2$	holding cost per unit in O.W.
$s$	shortage cost per unit
$l$	lost sale cost per unit
$S$	total ordered quantity
$d_1$	deterioration cost per unit in R.W.
$d_2$	deterioration cost per unit in O.W.

### 3. MATHEMATICAL MODELLING

Here in this paper a deterministic inventory model for deteriorating products with stock level dependent demand and shortages has been developed. At  $t=0$ , the  $S$  units of inventory enters in the system, out of which  $W$  units are kept in O.W. and the remaining  $(S-W)$  units are kept in R.W. First the units stocked in R.W. is consumed. During this time period the inventory in O.W. depletes due to the effect of deterioration only. When the inventory in R.W. becomes zero, only then the stock in O.W. is used to satisfy the customer needs. In O.W. the inventory level becomes zero at  $t=v$  after satisfying the customer's needs and due to deterioration. After  $t=v$ , the shortages occur and the occurring shortages are partially backlogged at a constant rate.

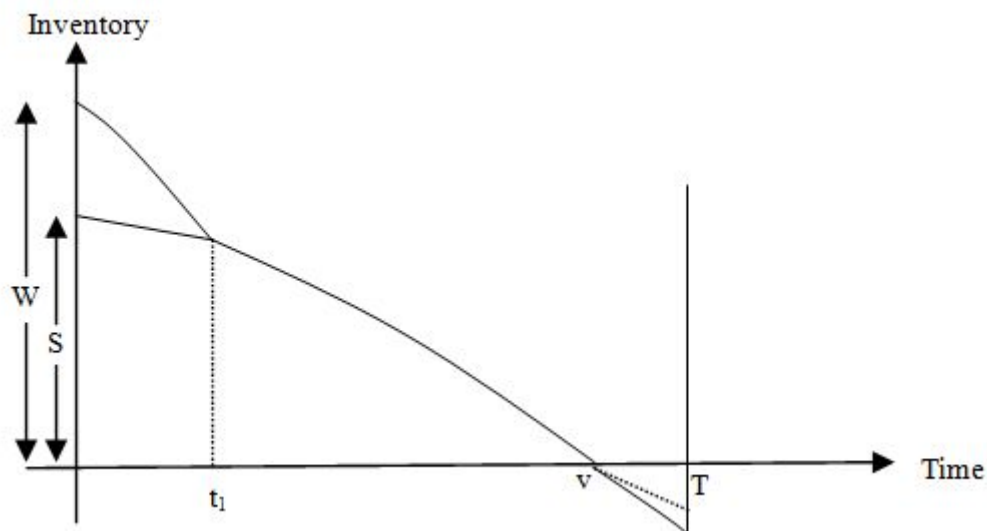


Fig. 1: Inventory time behavior in O.W. and R.W

The differential equations showing the inventory time behavior of the system are given as follow:

$$\frac{dI_r(t)}{dt} = -(\alpha + \beta W) - \theta I_r(t) \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI_0(t)}{dt} = -\theta I_0(t) \quad 0 \leq t \leq t_1 \quad (2)$$

$$\frac{dI_0(t)}{dt} = -\theta I_0(t) - (\alpha + \beta I_0(t)) \quad t_1 \leq t \leq v \quad (3)$$

$$\frac{dI_0(t)}{dt} = -\alpha \quad v \leq t \leq T \quad (4)$$

with boundary conditions:

$$I_r(t_1) = 0, \quad I_0(0) = W \quad \text{and} \quad I_0(v) = 0 \quad (5)$$



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Solving these equations with the help of these boundary conditions:

$$I_r(t) = (\alpha + \beta W)[(t_1 - t) + \frac{\theta}{6}(t_1^3 - t^3)]e^{-\frac{\theta t^2}{2}} \quad 0 \leq t \leq t_1 \quad (6)$$

$$I_0(t) = We^{-\frac{\theta t^2}{2}} \quad 0 \leq t \leq t_1 \quad (7)$$

$$I_0(t) = \alpha[(v - t) + \frac{\beta}{2}(v^2 - t^2) + \frac{\theta}{6}(v^3 - t^3)]e^{-\beta t - \frac{\theta t^2}{2}} \quad t_1 \leq t \leq v \quad (8)$$

$$I_0(t) = \alpha(v - t) \quad v \leq t \leq T \quad (9)$$

### Different Associated Cost:

#### i. Ordering Cost:

Since the order is given at the beginning of each cycle, so the ordering cost is given by:

$$O.C. = A \quad (10)$$

#### ii. Holding Cost:

##### (a) Holding Cost in R.W.:

Holding cost per cycle in rented warehouse is given as follow:

$$H.C_R = h_1 \int_0^{t_1} I_r(t) dt$$

$$H.C_R = h_1(\alpha + \beta W)\left(\frac{t_1^2}{2} + \frac{\theta}{12}t_1^4\right) \quad (11)$$

##### (b) Holding Cost in O.W.:

Holding cost per cycle in owned warehouse can be calculated as follow:

$$H.C_o = h_2 \int_0^{t_1} I_0(t) dt + h_2 \int_{t_1}^v I_0(t) dt$$

$$H.C_o = h_2 W \left( t_1 - \frac{\theta}{6} t_1^3 \right) + h_2 \alpha \left\{ \frac{v^2}{2} + \frac{\beta}{6} v^3 + \frac{\theta}{12} v^4 \right\} - h_2 \alpha \left\{ \left( vt_1 - \frac{t_1^2}{2} \right) + \frac{\beta}{2} \left( v^2 t_1 - \frac{t_1^3}{3} \right) + \frac{\theta}{6} \left( v^3 t_1 - \frac{t_1^4}{4} \right) - \beta \left( \frac{vt_1^2}{2} - \frac{t_1^3}{3} \right) - \frac{\theta}{2} \left( \frac{vt_1^3}{3} - \frac{t_1^4}{4} \right) \right\} \quad (12)$$

#### iii. Shortage Cost:

The shortage occurs only in owned warehouse. So the shortage cost can be calculated as:

$$S.C. = s \int_v^T \alpha dt$$

$$S.C. = s\alpha(T - v) \quad (13)$$

#### iv. Lost Sale Cost:

During stock out a certain ratio of customers come back to make their purchases and the remaining other customers go to other to fulfill their needs. So the lost sale cost in this case will be:

$$L.S.C. = l \int_v^T (1 - \theta) \alpha dt$$

$$L.S.C. = l\alpha(1 - \theta)(T - v) \quad (14)$$



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v. **Deterioration Cost:**

(a) **Deterioration cost in R.W.:**

$$D.C._R = I_r(0) - \int_0^{t_1} D dt$$

$$D.C._R = d_1 t_1^3 \frac{\theta}{6} (\alpha + \beta W) \quad (15)$$

(b) **Deterioration cost in O.W.:**

$$D.C._O = d_2 \left\{ W - \int_{t_1}^v D(t) dt \right\}$$

$$D.C._O = d_2 \left\{ W - \left\{ \alpha v + \beta \alpha \left( \frac{v^2}{2} + \frac{\beta}{6} v^3 + \frac{\theta}{8} v^4 \right) \right\} + \alpha t_1 + \beta \alpha \left\{ \left( v t_1 - \frac{t_1^2}{2} \right) + \frac{\beta}{2} \left( v^2 t_1 - \frac{t_1^3}{3} \right) + \frac{\theta}{6} \left( v^3 t_1 - \frac{t_1^4}{4} \right) - \beta \left( v \frac{t_1^2}{2} - \frac{t_1^3}{3} \right) \right\} \right\} \quad (16)$$

Therefore the T.A.C. of the system in this case will be:

$$T.A.C(t_1, v) = \frac{1}{T} [O.C. + H.C._R + H.C._O + S.C. + L.S.C. + D.C._O + D.C._R]$$

$$T.A.C(t_1, v) = A + h_1 (\alpha + \beta W) \left( \frac{t_1^2}{2} + \frac{\theta}{12} t_1^4 \right) + h_2 W \left( t_1 - \frac{\theta}{6} t_1^3 \right) + h_2 \alpha \left\{ \frac{v^2}{2} + \frac{\beta}{6} v^3 + \frac{\theta}{12} v^4 \right\}$$

$$- h_2 \alpha \left\{ \left( v t_1 - \frac{t_1^2}{2} \right) + \frac{\beta}{2} \left( v^2 t_1 - \frac{t_1^3}{3} \right) + \frac{\theta}{6} \left( v^3 t_1 - \frac{t_1^4}{4} \right) - \beta \left( \frac{v t_1^2}{2} - \frac{t_1^3}{3} \right) - \frac{\theta}{2} \left( \frac{v t_1^3}{3} - \frac{t_1^4}{4} \right) \right\}$$

$$+ s \alpha (T - v) + l \alpha (1 - \theta) (T - v) + d_1 t_1^3 \frac{\theta}{6} (\alpha + \beta W)$$

$$+ d_2 \left\{ W - \left\{ \alpha v + \beta \alpha \left( \frac{v^2}{2} + \frac{\beta}{6} v^3 + \frac{\theta}{8} v^4 \right) \right\} + \alpha t_1 + \beta \alpha \left\{ \left( v t_1 - \frac{t_1^2}{2} \right) + \frac{\beta}{2} \left( v^2 t_1 - \frac{t_1^3}{3} \right) + \frac{\theta}{6} \left( v^3 t_1 - \frac{t_1^4}{4} \right) - \beta \left( v \frac{t_1^2}{2} - \frac{t_1^3}{3} \right) \right\} \right\} \quad (17)$$

Now to minimize the T.A.C. we have to find out the optimal values of  $t_1$  and  $v$ , which can be obtained by solving the following equations simultaneously:

$$\frac{\partial T.C(t_1, v)}{\partial t_1} = 0, \text{ and } \frac{\partial T.C(t_1, v)}{\partial v} = 0 \quad (18)$$

These are the sufficient conditions:

$$\frac{\partial T.C^2(t_1, v)}{\partial t_1^2} \Big|_{t_1^*, v^*} > 0, \text{ and } \frac{\partial T.C(t_1, v)}{\partial v} \Big|_{t_1^*, v^*} > 0 \quad (19)$$

$$\text{and } \left( \frac{\partial^2 T.C_1(t_1, v)}{\partial t_1^2} \right) \left( \frac{\partial^2 T.C_1(t_1, v)}{\partial v^2} \right) - \left( \frac{\partial^2 T.C_1(t_1, v)}{\partial t_1 \partial v} \right)^2 \Big|_{(t_1^*, v^*)} > 0$$

(20)



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## 4. NUMERICAL EXAMPLE

$s = 4rs/unit$ ,  $A = 500rs/order$ ,  $\alpha = 250units$ ,  $\beta = 0.02$ ,  $\theta = 0.001$ ,  $d_1 = 14rs/unit$ ,  
 $d_2 = 14rs/unit$ ,  $h_1 = 0.6rs/unit$ ,  $h_2 = 0.5rs/unit$ ,  $T = 120days$ ,  $W = 500units$ ,  $l = 5rs/unit$   
 Corresponding to these values the optimal values are as follows:  
 $t_1 = 83.6576days$ ,  $v = 102.45days$ ,  $T.A.C. = 12350.2rs$

### Sensitivity Analysis

Table 1: Variation in T.A.C. with the variation in  $\alpha$ :

% variation in $\alpha$	$\alpha$	$t$	$i$	$v$	T . A . C .
- 2 0 %	2 0 0	8 3 . 6 5 7 5		1 0 2 . 4 5	9 9 8 5 . 4 8
- 1 5 %	2 1 2 . 5	8 3 . 6 5 7 5		1 0 2 . 4 5	1 0 5 7 6 . 7
- 1 0 %	2 2 5	8 3 . 6 5 7 5		1 0 2 . 4 5	1 1 1 6 7 . 9
- 5 %	2 3 7 . 5	8 3 . 6 5 7 5		1 0 2 . 4 5	1 1 7 5 9 . 1
0 %	2 5 0	8 3 . 6 5 7 5		1 0 2 . 4 5	1 2 3 5 0 . 2
5 %	2 6 2 . 5	8 3 . 6 5 7 5		1 0 2 . 4 5	1 2 9 4 1 . 4
1 0 %	2 7 5	8 3 . 6 5 7 5		1 0 2 . 4 5	1 3 5 3 2 . 6
1 5 %	2 8 7 . 5	8 3 . 6 5 7 5		1 0 2 . 4 5	1 4 1 2 3 . 8
2 0 %	3 0 0	8 3 . 6 5 7 5		1 0 2 . 4 5	1 4 7 1 5

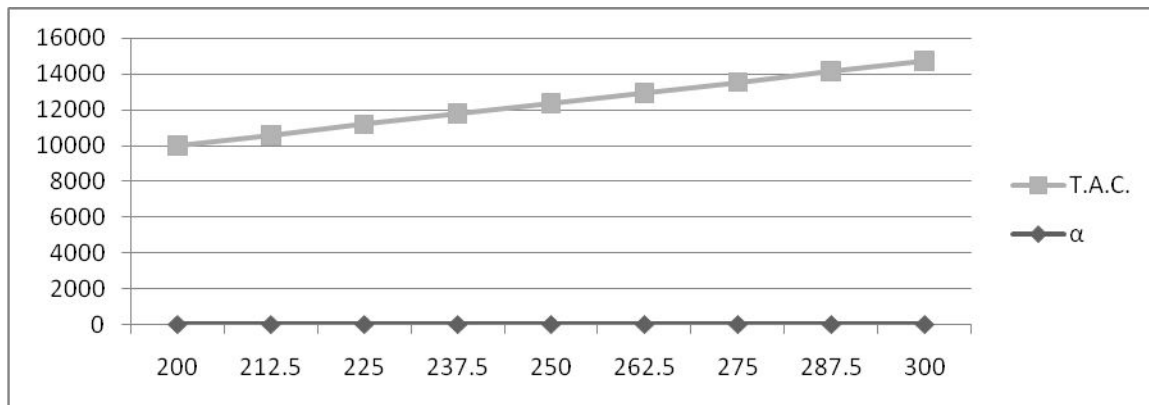


Fig 2: T.A.C. v/s  $\alpha$

Table 2: Variation in T.A.C. with the variation in  $\beta$ :

% variation in $\beta$	$\beta$	$t$	$i$	$v$	T . A . C .
- 2 0 %	0 . 0 1 6	8 9 . 2 2 0 4		1 0 9 . 3 4 5	1 4 8 9 4 . 3
- 1 5 %	0 . 0 1 7	8 7 . 7 1 3 1		1 0 7 . 4 7 7	1 4 1 6 3 . 9
- 1 0 %	0 . 0 1 8	8 6 . 2 8 8 2		1 0 5 . 7 1 1	1 3 5 0 2
- 5 %	0 . 0 1 9	8 4 . 9 3 8 5		1 0 4 . 0 3 8	1 2 8 9 9 . 9
0 %	0 . 0 2	8 3 . 6 5 7 5		1 0 2 . 4 5	1 2 3 5 0 . 2
5 %	0 . 0 2 1	8 2 . 4 3 9 6		1 0 0 . 9 4 1	1 1 8 4 6 . 8
1 0 %	0 . 0 2 2	8 1 . 2 7 9 7		9 9 . 5 0 3	1 1 3 8 4 . 3
1 5 %	0 . 0 2 3	8 0 . 1 7 3 4		9 8 . 1 3 1 9	1 0 9 5 8 . 3



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2	0	%	0 . 0 2 4	7 9 . 1 1 6 7	9 6 . 8 2 2 1	1 0 5 6 4 . 8
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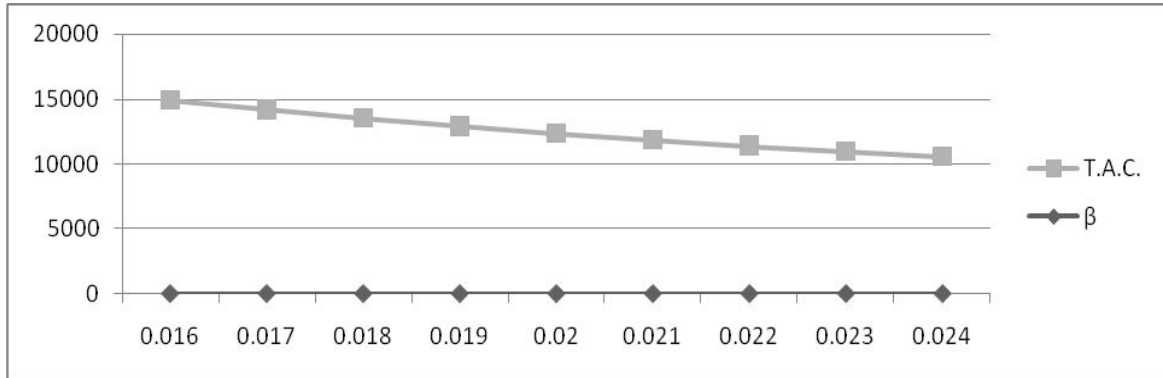


Fig. 3: T.A.C. v/s  $\beta$

Table 3: Variation in T.A.C. with the variation in  $\theta$ :

% variation in $\theta$			$\theta$	t	v	T . A . C .
-	2	0	0 . 0 0 0 8	9 0 . 1 6 6 6	1 1 0 . 5 1 8	1 1 6 5 9 . 1
-	1	5	0 . 0 0 0 8 5	8 8 . 3 4 7 9	1 0 8 . 2 6 4	1 1 8 1 3 . 3
-	1	0	0 . 0 0 0 9	8 6 . 6 6 7 9	1 0 6 . 1 8 1	1 1 9 7 8 . 9
-		5	0 . 0 0 0 9 5	8 5 . 1 0 9 3	1 0 4 . 2 5	1 2 1 5 7 . 4
0			0 . 0 0 1	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 3 5 0 . 2
5			0 . 0 0 1 0 5	8 2 . 3 0 0 4	1 0 0 . 7 6 8	1 2 5 5 9 . 5
1	0		0 . 0 0 1 1	8 1 . 0 2 7 8	9 9 . 1 9 0 8	1 2 7 8 7 . 8
1	5		0 . 0 0 1 1 5	7 9 . 8 3 0 9	9 7 . 7 0 7 3	1 3 0 3 7 . 6
2	0		0 . 0 0 1 2	7 8 . 7 0 2 2	9 6 . 3 0 8 3	1 3 3 1 2 . 8

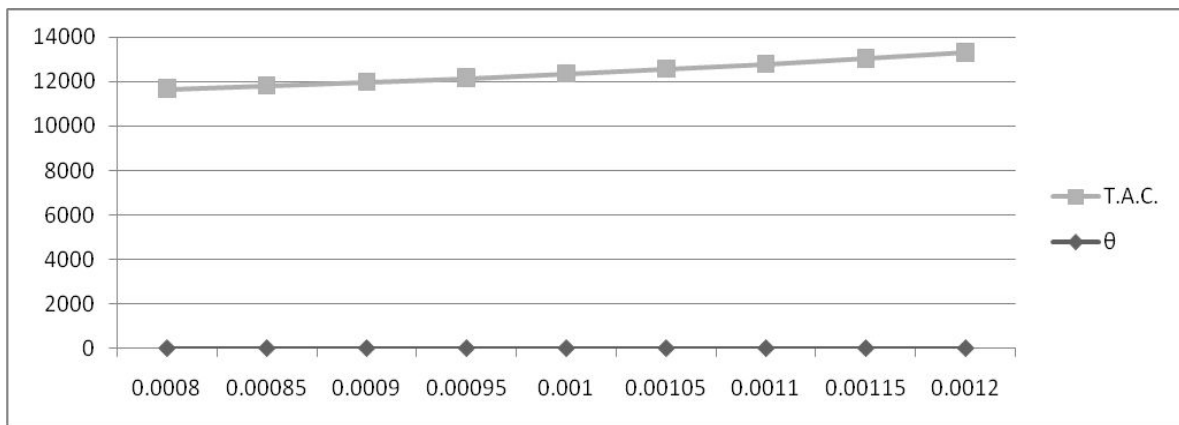


Fig. 4: T.A.C. v/s  $\theta$

Table 4: Variation in T.A.C. with the variation in W:

% variation in W			W	t	v	T . A . C .
-	2	0	4 0 0	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 2 4 5 . 8
-	1	5	4 2 5	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 2 7 1 . 9



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-	1	0	%	4	5	0	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 2 9 8
-	5		%	4	7	5	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 3 2 4 . 1
0			%	5	0	0	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 3 5 0 . 2
5			%	5	2	5	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 3 7 6 . 4
1	0		%	5	5	0	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 4 0 2 . 5
1	5		%	5	7	5	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 4 2 8 . 6
2	0		%	6	0	0	8 3 . 6 5 7 5	1 0 2 . 4 5	1 2 4 5 4 . 7

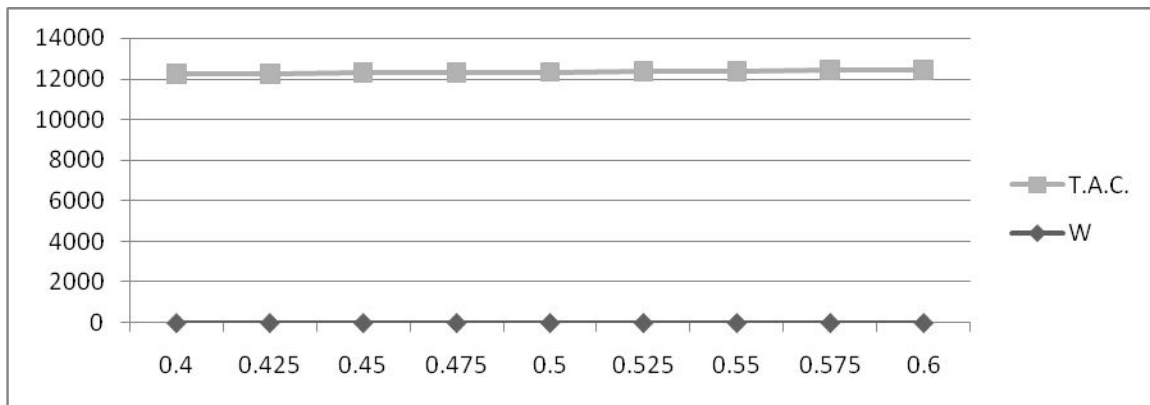


Fig. 5: T.A.C. w/s W

5. OBSERVATIONS

A sensitivity analysis is carried out with respect to different system parameters taking one at a time. Table 1 shows that when the demand parameter  $\alpha$  increases, then due to the increment in initial demand rate, purchasing cost of the system increases and it results also an increment in T.A.C. of the system. But when the demand parameter  $\beta$  increases, the stock level dependent factor of demand increases and in this case it is observed that for a fix level of stock level the demand increases and due to this the profit will increase and T.A.C. of the system will decrease. Table 3 shows the variation in deterioration parameter  $\theta$  by -20%, -15%, -10%, -5%, 0%, 5%, 10%, 15%, 20%. As the value of  $\theta$  increases then due to the increases product deterioration the T.A.C. of the system continuously increases. Table 4 lists the changes in owned warehouse capacity W. From this table it is observed that with the increased capacity of owned warehouse the value of deterioration cost and holding cost increases and it results an increment in T.A.C. So it will be beneficial to rent a warehouse except to increase the owned warehouse capacity. All of these variation are also shown in figure 2-5.

6. CONCLUSION

Here in this paper an inventory model for deteriorating products with stock dependent demand has been developed. It is shown in this paper that the demand for the products increases with the stock level but after a certain stock level it becomes constant. The inventory holding cost is different in both the warehouse due to different preservation facility. The shortages are allowed in owned warehouse and occurring shortages are partially backlogged and partially lost. It is observed that an increased capacity of owned warehouse also increases the cost of the system and the increases demand due to high stock level decreases the total system cost. This model reflects some realistic feature and so it is useful in retail business.

For the future research scope this model can be extended for trade credit period, inflation and more variable rates of demand pattern

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